

Dynamics of gap solitons in a dipolar Bose-Einstein condensate on a three-dimensional optical lattice

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Abstract. We suggest and study the stable disk- and cigar-shaped gap solitons of a dipolar Bose-Einstein condensate of ^{52}Cr atoms localized in the lowest band gap by three optical-lattice (OL) potentials along orthogonal directions. The one-dimensional version of these solitons of experimental interest confined by an OL along the dipole moment direction and harmonic traps in transverse directions is also considered. Important dynamics of (i) breathing oscillation of a gap soliton upon perturbation and (ii) dragging of a gap soliton by a moving lattice along axial z direction demonstrates the stability of gap solitons. A movie clip of dragging of three-dimensional gap soliton is included.

PACS numbers: 67.85.Hj, 03.75.Lm, 03.75.Nt

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Solitons in Bose-Einstein condensate (BEC) have drawn a great deal of attention recently. Experimentally, bright solitons were created in attractive BEC of ^7Li [1] and ^{85}Rb [2] atoms resulting from a cancellation of nonlinear attraction and linear dispersive effects. For a repulsive BEC, dark solitons have also been experimentally observed [3]. In a repulsive BEC on a periodic optical-lattice (OL) localized (dynamically) stable states of gap solitons in the band gap can be made [4, 5]. An OL simulates the periodic electron-atom potential in a solid, and the study of gap solitons in OL and the related band-gap structure in the presence of nonlinear interaction is also of interest in condensed-matter physics [6]. The mathematical model of gap solitons and atomic band-gap in a BEC is identical to that of light propagation in photonic crystals with cubic nonlinearity in presence of photonic band-gap and this makes the study of gap solitons in a BEC of general interest [4]. Later, the possibility of gap solitons in Fermi superfluid has been suggested [7]. Gap solitons have been observed in photonic crystals [8]. A gap soliton of about 250 ^{87}Rb atoms in a nearly one-dimensional (1D) OL [9, 10] has been created experimentally. In the experiment, the OL was subjected to acceleration, in order to push atoms into a gap soliton state [9]. Other possibility for the creation of gap solitons may employ the phase-imprinting method [11].

The alkali metal atoms used in BEC experiments have negligible dipole moment. However, most bosonic atoms and molecules have large dipole moments and a ^{52}Cr BEC [12], with a larger long-range dipolar interaction superposed on the short-range atomic interaction, has been realized. Thus one can study a dipolar BEC (DBEC) with variable short-range interaction [12] using a Feshbach resonance [13]. The stability of a DBEC depends not only on the atomic interaction, but also on trap geometry [12, 14, 15]. In a disk configuration the dipolar interaction is repulsive and a DBEC is more stable; while, in a cigar configuration the dipolar interaction is attractive and a DBEC is less stable due to collapse instability [12, 16]. The controllable short-range interaction together with the dipolar interaction makes the DBEC an attractive system for experimental soliton generation and a challenging system for theoretical investigation. There have already been studies of bright solitons in DBEC [17].

Using the numerical and variational solution of the GP equation we predict and study stationary and dynamical properties of three-dimensional (3D) DBEC gap solitons, of both cigar and disk shapes, localized in the lowest band gap by three orthogonal OL potentials. The BEC gap solitons are realizable only for all repulsive atomic interactions below a maximum value so that the chemical potential falls in the band gap. The DBEC gap solitons in disk shape can also be formed for weakly attractive short-range interaction. The cigar-shaped DBEC gap solitons are formed only for short-range repulsion above a limiting value. We also consider the one-dimensional gap solitons of direct experimental interest confined by an OL along the dipole moment direction and harmonic traps in transverse directions.

We numerically explore the breathing oscillation of the gap solitons upon a small perturbation. We illustrate the dragging of the gap solitons by an OL moving along the axial z direction [18, 19]. These solitons can be dragged without deformation for a

reasonably large velocity.

Here we study the DBEC gap soliton of N atoms, each of mass m , using the dimensionless GP equation [12]

$$i \frac{\partial \phi(\mathbf{r}, t)}{\partial t} = \left[-\frac{1}{2} \nabla^2 + V(\mathbf{r}) + 4\pi a N |\phi(\mathbf{r}, t)|^2 + N \int U_{dd}(\mathbf{r} - \mathbf{r}') |\phi(\mathbf{r}', t)|^2 d\mathbf{r}' \right] \phi(\mathbf{r}, t), \quad (1)$$

with dipolar interaction $U_{dd}(\mathbf{R}) = 3a_{dd}(1 - 3\cos^2\theta)/R^3$, $\mathbf{R} = \mathbf{r} - \mathbf{r}'$. Here $V(\mathbf{r})$ is the confining potential, $\phi(\mathbf{r}, t)$ the wave function at time t with normalization $\int |\phi(\mathbf{r}, t)|^2 d\mathbf{r} = 1$, a the atomic scattering length, θ the angle between \mathbf{R} and the polarization direction z . The constant $a_{dd} = \mu_0 \bar{\mu}^2 m / (12\pi \hbar^2)$ is a length characterizing the strength of dipolar interaction and its experimental value for ^{52}Cr is $15a_0$ [12], with a_0 the Bohr radius, $\bar{\mu}$ the (magnetic) dipole moment of a single atom, and μ_0 the permeability of free space. The OL potential in a specific direction, say \hat{z} , is $V(\hat{z}) = s_z E_R \sin^2(2\pi \hat{z}/\lambda)$, where $E_R = \hbar^2/(2m\lambda^2)$ is the recoil energy of an atom, λ is the wave length of the laser and s_z is the strength of the OL. In (1) length is measured in units of $\lambda/2\pi$ (taken here as $1 \mu\text{m}$ for a far infrared laser), time t in units of $m\lambda^2/2\pi\hbar$, and energy in units of $\hbar^2/m\lambda^2$. The dimensionless 3D periodic OL trap can now be written as

$$V(\mathbf{r}) = -[V_\rho \{\cos(2x) + \cos(2y)\} + V_z \cos(2z)], \quad (2)$$

where the parameters V_ρ and V_z are the strengths of the OL's in radial and axial directions and can be varied to achieve the disk- ($V_z > V_\rho$) and cigar-shaped ($V_\rho > V_z$) gap solitons in DBEC.

The gap solitons considered here predominantly have a Gaussian shape and for these solitons a Gaussian variational solution is known to lead to a good description [7]. The variational approximation provides an analytical understanding and also yields interesting results when the numerical procedure is difficult to implement. A better but more complicated description can be obtained with a different ansatz of variational functions [20]. The Lagrangian density of (1) is given by

$$\mathcal{L} = \frac{i}{2} (\phi \phi_t^* - \phi^* \phi_t) + \frac{1}{2} |\nabla \phi|^2 + V(\mathbf{r}) |\phi|^2 + 2\pi a N |\phi|^4 + \frac{N}{2} |\phi|^2 \int U_{dd}(\mathbf{r} - \mathbf{r}') |\phi(\mathbf{r}')|^2 d\mathbf{r}'. \quad (3)$$

We use the Gaussian ansatz [21]

$$\phi(\mathbf{r}, t) = \frac{\pi^{-3/4}}{w_\rho \sqrt{w_z}} \exp \left(-\frac{\rho^2}{2w_\rho^2} - \frac{z^2}{2w_z^2} + i\alpha \rho^2 + i\beta z^2 \right) \quad (4)$$

for a variational calculation, where w_ρ and w_z are time-dependent radial and axial widths, and α and β time-dependent phases. The effective Lagrangian $L \equiv \int \mathcal{L} d\mathbf{r}$ (per particle) becomes

$$L = (w_\rho^2 \dot{\alpha} + w_z^2 \dot{\beta}/2) - 2V_\rho \exp(-w_\rho^2) - V_z \exp(-w_z^2)$$

$$\begin{aligned}
& + 1/(2w_\rho^2) + 1/(4w_z^2) + 2w_\rho^2\alpha^2 + w_z^2\beta^2 \\
& + Na_{dd}/(\sqrt{2\pi}w_\rho^2w_z)[a/a_{dd} - f(\kappa)],
\end{aligned} \tag{5}$$

with

$$f(\kappa) = [1 + 2\kappa^2 - 3\kappa^2d(\kappa)]/(1 - \kappa^2), \tag{6}$$

$$d(\kappa) = \frac{\text{atanh}\sqrt{1 - \kappa^2}}{\sqrt{1 - \kappa^2}}, \quad \kappa = \frac{w_\rho}{w_z}. \tag{7}$$

The Euler-Lagrange equations for parameters w_ρ, w_z, α and β yield the following equations for widths w_ρ and w_z

$$\ddot{w}_\rho + \frac{4V_\rho w_\rho}{e^{w_\rho^2}} = \frac{1}{w_\rho^3} + \frac{N}{\sqrt{2\pi}} \frac{[2a - a_{dd}g(\kappa)]}{w_\rho^3 w_z}, \tag{8}$$

$$\ddot{w}_z + \frac{4V_z w_z}{e^{w_z^2}} = \frac{1}{w_z^3} + \frac{2N}{\sqrt{2\pi}} \frac{[a - a_{dd}h(\kappa)]}{w_\rho^2 w_z^2}, \tag{9}$$

with

$$g(\kappa) = [2 - 7\kappa^2 - 4\kappa^4 + 9\kappa^4d(\kappa)]/(1 - \kappa^2)^2, \tag{10}$$

$$h(\kappa) = [1 + 10\kappa^2 - 2\kappa^4 - 9\kappa^2d(\kappa)]/(1 - \kappa^2)^2. \tag{11}$$

The chemical potential μ is given by

$$\begin{aligned}
\mu &= \frac{1}{2w_\rho^2} + \frac{1}{4w_z^2} + \frac{2N[a - a_{dd}f(\kappa)]}{\sqrt{2\pi}w_zw_\rho^2} \\
&\quad - 2V_\rho \exp(-w_\rho^2) - V_z \exp(-w_z^2).
\end{aligned} \tag{12}$$

For gap solitons the system must be repulsive. For a normal BEC ($a_{dd} = 0$), attraction corresponds to $a < 0$ and repulsion to $a > 0$ and gap solitons are formed below a limiting repulsive scattering length $0 < a < a_c$. For $a_{dd} > 0$, in the disk shape, the dipole moment contributes repulsively and due to this extra repulsion a DBEC gap soliton can be formed in a window of scattering lengths $-a_1 < a < a_2$ between a limiting attractive ($-a_1$) and repulsive (a_2) limits. However, in a weak cigar shape, the dipole moment contributes attractively and due to the extra attraction a DBEC gap soliton can be formed in a window of repulsive scattering lengths $a_3 < a < a_4$. The limiting values a_1, a_2, a_3 and a_4 can be obtained from a solution of the GP equation. In a strong cigar regime, the dipole moment contributes to strong attraction and no gap soliton can be formed due to collapse instability.

The cigar-shaped quasi-1D gap solitons with a radial harmonic trap of frequency Ω_ρ satisfy (1) with $V(\mathbf{r}) = \Omega_\rho^2 \rho^2/2 - V_z \cos(2z)$. In this case it is convenient to solve the 1D GP equation [22, 23] with the reduced dipolar interaction

$$\begin{aligned}
U_{dd}^{1D}(Z) &= 3a_{dd}N[4\delta(\sqrt{t})/3 + 2\sqrt{t} - \sqrt{\pi}(1 + 2t) \\
&\quad \times e^t \{1 - \text{Erf}(\sqrt{t})\}]/(\sqrt{2}d_\rho)^3,
\end{aligned} \tag{13}$$

where $Z = |z - z'|$, $d_\rho = 1/\sqrt{\Omega_\rho}$, $t = [Z/(\sqrt{2}d_\rho)]^2$ and where we have included the proper δ -function term mentioned in [22]. The 1D GP equation is given by

$$i \frac{\partial \phi_{1D}(z, t)}{\partial t} = \left[-\frac{1}{2} \frac{\partial^2}{\partial z^2} - V_z \cos(2z) + \frac{2aN}{d_\rho^2} |\phi_{1D}(z, t)|^2 \right]$$

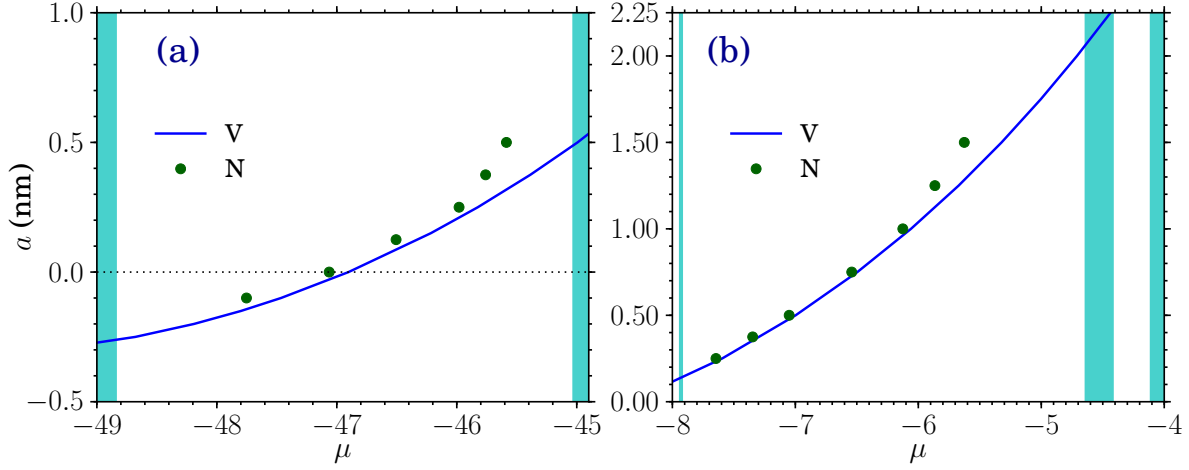


Figure 1. (Color online) Band (shaded area), gap (white area) and the numerical (N) and variational (V) $a - \mu$ plots of 3D DBEC gap solitons for $a_{dd} = 15a_0$, $N = 500$ and $V_\rho = 5$, (a) $V_z = 50$ (disk) and (b) $V_z = 4$ (cigar).

$$+ N \int U_{dd}^{1D}(Z) |\phi_{1D}(z', t)|^2 dz' \Big] \phi_{1D}(z, t), \quad (14)$$

Using the Gaussian ansatz

$$\phi_{1D}(z) = \frac{\pi^{-1/4}}{\sqrt{w_z}} \exp \left[-\frac{z^2}{2w_z^2} + i\beta z^2 \right], \quad (15)$$

the variational Lagrangian becomes

$$L = \frac{w_z^2 \dot{\beta}}{2} - V_z \exp(-w_z^2) + \frac{1}{4w_z^2} + w_z^2 \beta^2 + \frac{Na_{dd}}{\sqrt{2\pi}d_\rho^2 w_z} \left[\frac{a}{a_{dd}} - f(\kappa_0) \right]; \quad \kappa_0 = \frac{d_\rho}{w_z}. \quad (16)$$

The variational equation for w_z becomes

$$\ddot{w}_z + \frac{4V_z w_z}{\exp(w_z^2)} = \frac{1}{w_z^3} + \frac{2N[a - a_{dd}h(\kappa_0)]}{\sqrt{2\pi}w_z^2 d_\rho^2}. \quad (17)$$

The corresponding chemical potential is

$$\mu = \frac{1}{4w_z^2} + \frac{2N[a - a_{dd}f(\kappa_0)]}{\sqrt{2\pi}w_z d_\rho^2} - \frac{V_z}{\exp(w_z^2)}. \quad (18)$$

To study the gap solitons, we perform a 3D numerical simulation employing real-time propagation with Crank-Nicolson method [24]. The numerical solutions are obtained by averaging the wave function over iterations with the variational solution as input. The dipolar interaction is evaluated by fast Fourier transform [14].

The stationary gap solitons appear in the band gap of the OL and a prior knowledge of the band and gap of the periodic OL is of advantage. The dipolar interaction is most prominent in the cigar (attractive) and disk (repulsive) shapes and hence we study the gap solitons in these two shapes. The OL parameters in these cases are taken as (a)

$V_\rho = 5, V_z = 50$ (disk shape) and (b) $V_\rho = 5, V_z = 4$ (cigar shape). The band and gap in these cases are shown in figure 1 together with the variational and numerical chemical potential μ for the stable gap solitons in the lowest band gap as a function of scattering length a for a DBEC ($a_{dd} = 15a_0$). The gap solitons in the higher band gaps are found to be dynamically unstable. In figure 1, the agreement between variational and numerical results worsens for large a . The system is more repulsive for large a and tends to occupy multiple OL sites and a single-peak Gaussian variational ansatz may not provide a good approximation to reality. In the cigar shape ($V_z = 4$), the dipolar interaction is attractive and hence DBEC gap solitons are possible for $a \equiv a_3 > 0.1343$ nm as in figure 1 (b). In the disk shape ($V_z = 50$), the dipolar interaction is repulsive, and hence DBEC gap solitons are possible for $a \equiv -a_1 > -0.2635$ nm as in figure 1 (a).

In figure 2 we show the 3D contour density profiles of some typical stationary gap solitons with $N = 500, V_\rho = 5, a_{dd} = 15a_0$ for variable a and V_z . We show disk-shaped profiles for $V_z = 50$ and (a) $a = 0.5$ and (b) $a = -0.1$ and cigar-shaped profiles for $V_z = 4$ and (c) $a = 1$ and (d) $a = 0.5$. For both disk and cigar shapes, the increased scattering length implies more nonlinear repulsion and one can have more atoms in adjacent OL sites. Although the gap solitons in figure 2 occupy multiple OL sites, more than 95 % of the matter is contained in the central site and an approximate Gaussian distribution

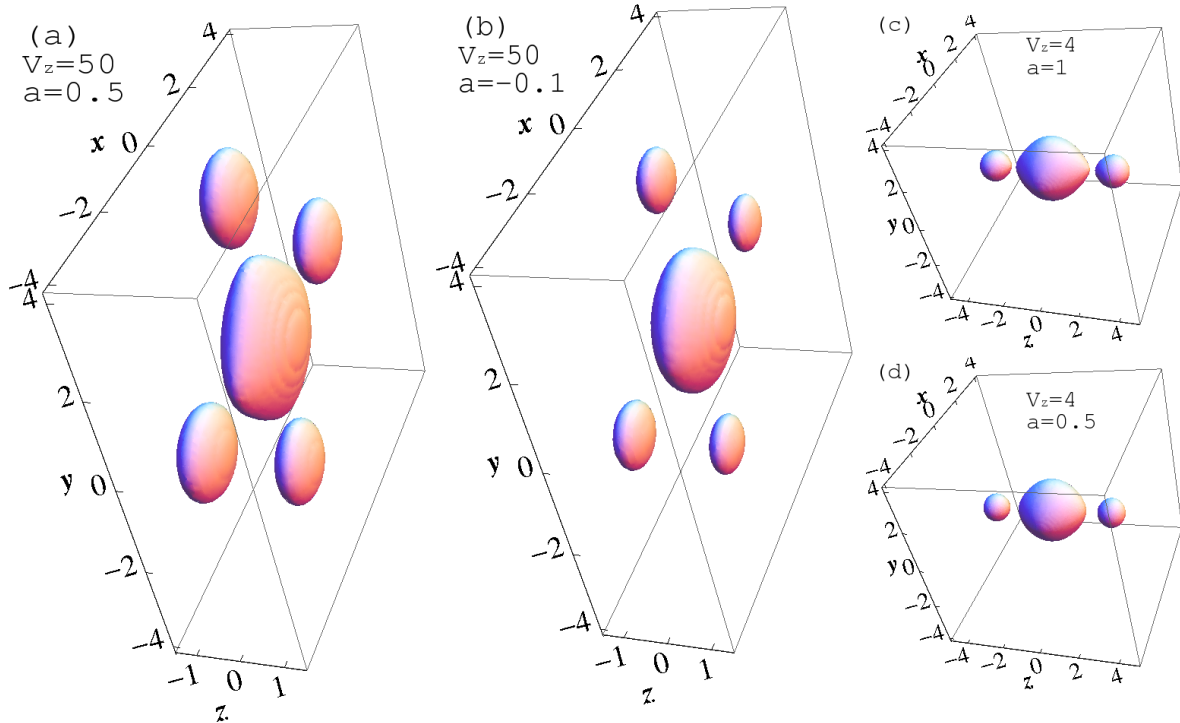


Figure 2. (Color online) 3D contour density plot of gap solitons for $N = 500, V_\rho = 5, a_{dd} = 15a_0$. (a) $V_z = 50, a = 0.5$, (disk), (b) $V_z = 50, a = -0.1$, (disk), (c) $V_z = 4, a = 1$, (cigar), (d) $V_z = 4, a = 0.5$, (cigar). The density at contour is 0.002.

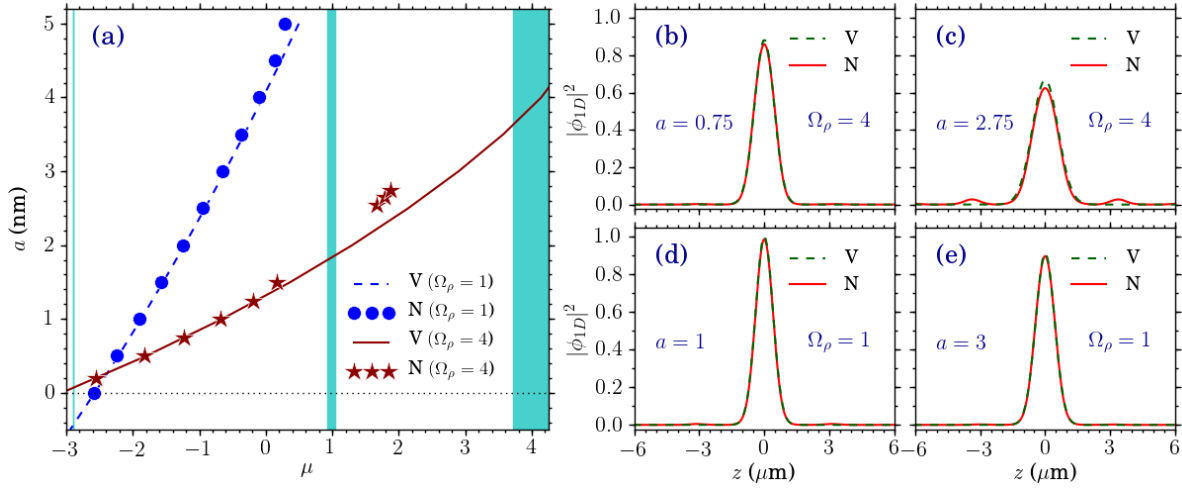


Figure 3. (Color online) (a) Band (shaded area), gap (white area) and the numerical (N) and variational (V) $a-\mu$ plot of 1D DBEC gap solitons for $\Omega_\rho = 1$ (disk) and $\Omega_\rho = 4$ (cigar). Variational and numerical densities of the 1D DBEC for (b) $\Omega_\rho = 4, a = 0.75$ nm, (c) $\Omega_\rho = 4, a = 2.75$ nm, (d) $\Omega_\rho = 1, a = 1$ nm, and (e) $\Omega_\rho = 1, a = 3$ nm. In all cases $a_{dd} = 15a_0$, $N = 500$ and $V_z = 5$.

is valid.

Next we study 1D gap solitons trapped by the OL $-V_z \cos(2z)$ along the z direction and harmonic trap $\Omega_\rho^2 \rho^2$ in transverse directions as calculated from numerical and variational solutions of the reduced 1D GP equation [22] with dipolar interaction (13). In figure 3 (a) we show the bands and gaps as well as the $a-\mu$ plot of the 1D gap solitons. For $\Omega_\rho = 1$, the DBEC is of disk shape and the contribution of the dipolar term is repulsive, and for $\Omega_\rho = 4$, DBEC is of cigar shape and the dipolar term contributes

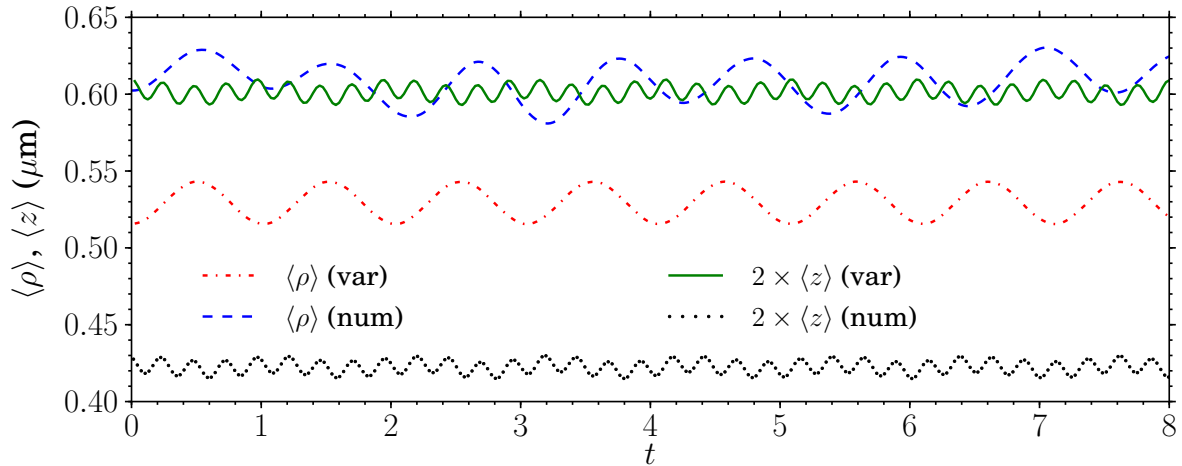


Figure 4. (Color online) Numerical (num) and variational (var) rms sizes $\langle \rho \rangle, \langle z \rangle$ during breathing oscillation of a disk-shaped 3D DBEC for $N = 500, a_{dd} = 15a_0, V_z = 50, V_\rho = 5$ as a is changed from 0 to 0.1.

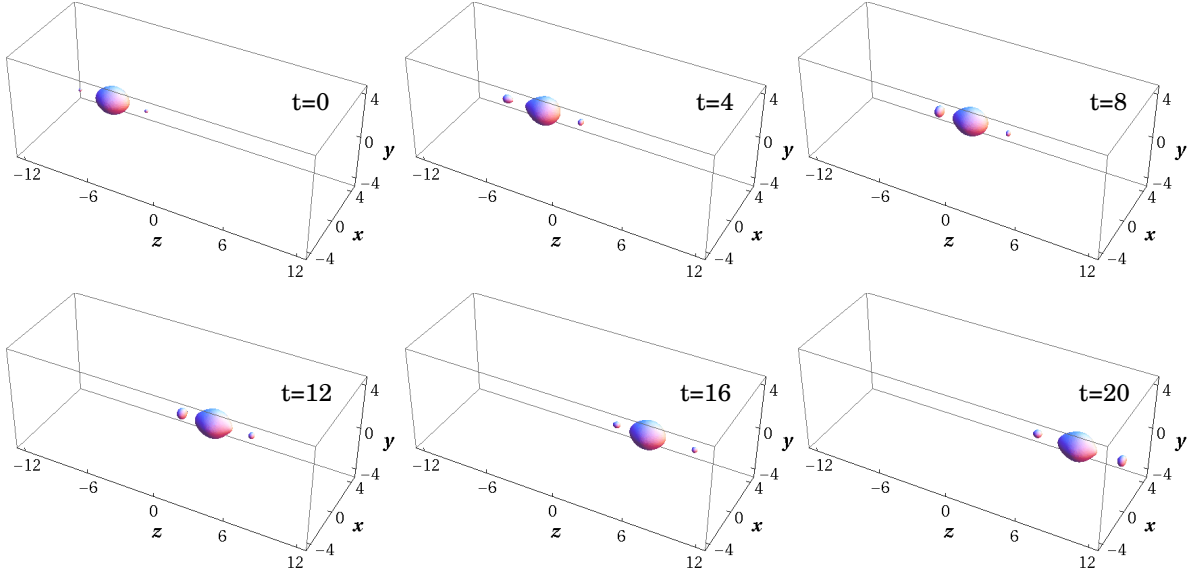


Figure 5. (Color online) Snapshots of 3D contour density profiles of a cigar-shaped gap soliton for $V_z = 4$, $V_\rho = 5$, $N = 500$, $a_{dd} = 15a_0$, and $a = 0.5$ nm during dragging by the moving OL $-V_\rho[\cos(2x) + \cos(2y)] - 2V_z \cos[2(z-vt)]$, $v = 0.75$ at times $t = 0, 4, 8, 12, 16$ and 20 .

attractively. In figures 3 (b) – (e) we plot the typical numerical 1D density profiles of some gap solitons and compare with variational results. Although, variational results exist across the bands, stable DBEC 3D gap solitons exist in the lowest band gap away from the band. In 1D some stable gap solitons are found in the first excited gap. Gap solitons cannot be stabilized close to the bands, e. g., near $\mu = -45$ and -49 in figure 1 (a), near $\mu = -4.5$ and -8 in figure 1 (b), and near $\mu = 1$ and 4 in figure 3. In figures 1 (a) and 3 (a) we clearly see that gap solitons are possible for small negative scattering lengths in disk-shaped DBEC and that, in the second band gap, 1D gap solitons can be stabilized only in a small domain of μ values between two bands.

To test the stability of the 3D DBEC gap solitons we now consider the breathing oscillation of a disk-shaped soliton initiated by slightly changing the scattering length a . Such a change in a can be made by varying a background magnetic field near a Feshbach resonance [13]. In figure 4 we plot the variational and numerical rms sizes $\langle \rho \rangle$, $\langle z \rangle$ versus time t . We find, using Fourier analysis, that the principal axial frequencies 1.0137 (variational) and 1.0812 (numerical) and the principal radial frequencies 0.2421 (variational) and 0.2469 (numerical) are in good agreement with each other.

Finally, we study the stability of a cigar-shaped 3D DBEC when dragged by an OL moving in the axial z direction. It has been experimentally found that a BEC confined by a transverse harmonic trap remains stable [19] when dragged by a moving 1D OL along the axial z direction below a critical velocity v_c of half the recoil velocity $[v_R = \hbar/(m\lambda)]$, which in present units ($\lambda = 2\pi$ and $m = \hbar = 1$) is $v_c = v_R/2 = 0.5$. Similar result is found to be true in the case of present 3D DBEC gap solitons. Steady

dragging is possible in this case even for velocities slightly larger than this limit. Instant snapshots of dragging dynamics with a velocity $v = 0.75 (< v_c = 1)$ along z direction is shown in figure 5 where we find that the gap soliton can be dragged without distortion with a reasonably large velocity for a considerable time. A movie of dragging of soliton of figure 5 showing stability is prepared and contained in supplementary clip S1 (see supplementary data available at stacks.iop.org/JPhysB/44/xxxxxx/mmedia, also available at <http://www.youtube.com/watch?v=wJbbKmsjQuU>).

To conclude, we suggested the possibility of 3D DBEC gap solitons of about 1000 Cr atoms confined in the lowest band gap by three OL in orthogonal directions and studied their statics (shape and chemical potential) and dynamics (breathing oscillation and dragging by an OL). In addition we studied 1D DBEC gap solitons using reduced 1D GP equations with a transverse harmonic trap and an axial OL along polarization direction. The 3D DBEC gap solitons are found to be dynamically stable during breathing oscillation and dragging for a long enough time for experiments and, with available technology, these solitons could be created and studied in laboratory. The present study opens up new directions of research that include, among others, excited states of gap solitons [4] and vortex gap solitons [25] in DBEC.

Acknowledgments

We thank FAPESP (Brazil), CNPq (Brazil), DST (India), and CSIR (India) for partial support.

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